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ABSTRACT

This study examined the development of young children's ability to solve linear patterning problems and investigated constructs of logical thinking which underlie the hierarachical order of the cognitive processes involved. A total of 252 children ranging in age from 4 through 7 years participated in the study. A range of ability and socioeconomic status levels were represented in this sample drawn from a suburban borough of a major Canadian metropolitan area. The instrument used consisted of 24 linear patterning tasks involving six cognitive processes and four pattern forms. Each subject was asked to use colored blocks to respond to six types of pattern recognition tasks: (1) exact reproduction of a pattern, (2) reproduction of a pattern with different stimuli of the same colors, (3) identification of a pattern from among three options, (4) extension of a pattern, (5) filling in missing portions of a pattern, and (6) reproduction of a pattern with the same stimuli of different colors. Results of a factor analysis indicated the existence of three strategy-oriented constructs underlying pattern task performance. (JMB)

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Development of Pattern Recognition in

Young Children

Abstract:

The objectives were to investigate:

a. development of linear patterning in young children as a hierarchy of cognitive processes, and

b. constructs underlying the hierarchical order.

Children four through seven representing a range of ability and SES levels were chosen in a suburban borough of a major Canadian metropolitan area. Each subject responded to twenty-four pattern recognition tasks using colored blocks: six cognitive processes each involving four pattern forms.

Factor analysis indicated the existence of three strategy-oriented constructs underlying pattern task performance. The hypothesized hierarchy of cognitive processes was sustained.

Development of Pattern Recognition in Young Children[1].

Introduction:

This study has been prompted by the recent growth of early childhood education and the inherent need for further research relating to young children in school learning settings. Some mathematics educators and psychologists have expressed concern over the educational inferences drawn from non-educational research and theory (Sullivan, 1967; Weaver, 1972). The influence of Piaget's research and theory in the design of curriculum and early childhood programs (Evans, 1971; Hooper & DeFrain, 1974) is evidence of the current interest in a concern for child development as a key to sound educational practice. Some mathematics educators and psychologists have expressed concern for the educational inferences drawn from non-educational research and theory (Sullivan, 1967; Weaver, 1972). The creation of developmentally appropriate learning situations remains paramount.

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1. This paper is based on doctoral studies pursued under Dr. V. J. Glennon at the University of Connecticut, 1976.

On the contention that pattern recognition ability is basic to mathematical insight (Gibb & Casteneda, 1975), is necessary to arithmetic functioning (Lovell, 1972), and provides early problem solving experiences (Nelson & Kirkpatrick, 1975), pattern recognition tasks have become well established as pre-number curricular experiences in pre-school, kindergarten, and primary level mathematics programs (Choat, 1973; Cielinski, 1967; Furth & Wachs, 1974; Gibb & Casteneda, 1975; Glennon, 1970, 1972; Heddens, 1974; McKillip, 1970; Nelson & Kirkpatrick, 1975; Payne, 1972; Uprichard, 1974).

The results of the investigation are intended to provide some means of gaining insight into the levels of thinking available to young children. Only through such knowledge can the concept of 'cognitive fit' (Cronbach, 1970; Skemp, 1971; Wapner, 1969) become a reality in classroom practice. "That which does not fit into [an existing schema] is largely not learned at all, and what is learnt temporarily is soon forgotten (Skemp, 1971, p. 43)."

Objectives:

The major purposes[1] of this investigation were:

1. to study the development of the ability of young children (ages four through seven) to solve problems in linear patterning as a hierarchy of cognitive processes, and
2. to investigate constructs of logical thinking underlying the hierarchical order of the processes.

A review of linear pattern tasks:

This review of the literature is concerned with the detailed nature of the several patterning tasks with which young subjects have been confronted in recent research projects. The purpose is twofold. One is to clarify what has become a confusion of terminology among researchers in the field. The second is to conjecture a theoretical hierarchy of the processes with regard to difficulty for young children. It will arise as a consequent of the necessarily intimate examination of the proportion of perceptual and logical thinking associated with each of the pattern processes.

(a) Reproduction: The term 'reproduction' as used by Burton (1973), Campbell (1973), Close (1976), Cromie (1971), Glennon and Alderman (1970), Rustigian (1976), and in the current study referred to the production of a model identical

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1. A study of the predictability of the variables: age, grade, general ability, and level of SES with respect to young children's pattern task achievement is to be the subject of a separate paper.

to a given left-right sequence and aligned directly in front of[1] that sequence in the same plane. (Example: ABCABC). In those cases in which only one repetend was presented or only one repetend was required of the subject, the process became precisely that of duplicating the order of a set of objects[2] (Braine, 1959; Huttenlocker, 1967; Sternberg, 1973). If more than one repetend were involved in a 'reproduction' task, (ABCABC) that task would differ from an order duplication problem only in that sets for ordering contain no repetition of objects whatever (Braine, 1959) (ABCDEF). In either case the task would require only one-to-one matching[3] of objects for the creation of a mirror image of the given model.

Huttenlocker (1967) reported that four- and five-year-olds found it easier to copy the order of a vertical set (stack) of objects than to copy a left-right array.[4] She agreed with Piaget and Inhelder (1967) regarding the increase in difficulty if the copy were to be made to one side of a

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1 Between the given sequence and the subject.

2 The concept involved is topological order (Piaget and Inhelder, 1967).

3 That is not to say that a subject who had acquired higher order operations would necessarily employ such a basic strategy. Rustigian (1976) recorded six distinct strategies used by subjects three- to five-years-old.

4 The experimenter constructed each model in front of the subject. McKillip (1970) did likewise.

given left-right array.[1] Furth and Wachs (1974) found that sequence reproduction became more complex as the separation between the blocks of the model was increased. In Braine's (1959) investigation of the age of order discrimination[2] he found that the length of the sequence (number of objects in the set) produced significant differences in performance (in three- to seven-year-olds). It should be noted that his subjects did not produce any sequences. They were required only to select a matching example from a multiple choice format.

(b) Identification: The term 'identification'[3] as used by Burton and others of that group of investigators and in the current study, referred to the matching of a given serial pattern to one of three possibilities.

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1 The process would then virtually be 'extension'.

2 Braine used a non-verbal protocol in an effort to reduce the age of order discrimination relative to that determined in the Genevan experiments.

3 There is a theoretical distinction between the identification of a sequence of objects in a given order and the identification of a sequence which displays a given pattern. In the former case, the identified sequence must be identical to the model (at least in terms of the attribute by which the ordering is defined). For example AB displays the reverse order of BA. However, AB is the same pattern form as BA (even though they also represent a pair of reversals). Similarly, ABB has the same pattern form as BAA. In the current study care was taken not to include such distractors among choice items (in deference to subjects able to abstract pattern concepts, the crux of the intra-variable transfer process).

Sternberg's (1974) 'original learning' concept also involved the same process with four options placed between the model and the subject rather than to the right of the model. It was the same concept which Braine (1959) used, but with four choices available through an essentially non-verbal technique involving a reinforcement factor.

A more general concept of identification as a form of response (through multiple choice selection) independent of the content of the (pattern) problem is also prevalent. Sternberg's (1973) investigation involved a variety of pattern processes but all the responses were in the identification format. Blackall (1975) included a 'select' process which involved an extrapolation[1] concept with subject selection of a completion. There were four choices in both cases.

Simon and Kotovsky (1963) pointed out that for serial pattern tasks, attainment is usually measured by ability to produce an object[2] "rather than the mere ability to identify an object as belonging to the concept" (p. 534). The discrepancy between an individual's reading vocabulary and speaking vocabulary was cited as an instance of the lack of

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1 'Pattern identification' for Rea and Reys (1970a) involved extrapolation.

2 The Thurstone Letter Series Completion Test is a case in point.

implication from ability to identify to ability to produce. Exceptions to this generalization were perhaps Braine's (1959) order discrimination tasks which involved the matching of up to seven element sequences. Producing such a sequence on a one-to-one basis would not require the same degree of cognition of order as would a selection from among several ready-made models involving subtle transpositions among adjacent pairs. Furth and Wachs (1974) found the difficulty of tasks based on response identification could be varied with spatial separation. As the distance from the model sequence to the choice alternatives was increased, the greater became the role of memory and recognition of the order relation at the expense of a one-to-one correspondence strategy.

(c) Interpolation: The term 'interpolation' as used by Close referred to the completion of a third repetend of a model in which one or more of the elements (other than the last one) were missing. (Example: ABCABC___A). In the current study, interpolation was restricted to a single element completion within any of the three repetends (excluding the initial and terminal elements), as in Blackall's (1975) investigation. (Example: ABC_BCABC). The former is closer to the more traditional[1] interpretation (Bartlett, 1958).

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1 Traditional interpolation involves the filling of a gap between any element of a sequence and the terminal element of a closed system (Bartlett, 1958). (Example: ABCAB____A).

It served as the basis of McKillip's (1970) "hidden blocks work sheets" (p. 18). The Nelson and Kirkpatrick (1975) problems for young children involved a combination of both concepts with single elements missing from non-contiguous positions. (Example: AB_A_CABC)

On the basis of a considerable study of the nature of thinking, Bartlett (1958) drew the following conclusions relative to interpolation:

(a) the probable number of distinct completions over "any fairly homogeneous social group" (p. 21) is less than the theoretically possible number and can only be established empirically,

(b) there exists a minimal amount of information which can be provided in a model so that there will be no two like completions of the gap and there also exists a maximal amount of information which can be provided so that "no normal person proceeding normally can fail to fill the gap in the same way as others"[1] (p. 21),

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1 The reference is to adults. The extent to which young children can do likewise has been investigated in the current study.

(c) the amount of information required for completion of a gap in the manner "most uniform throughout the operator's cultural group" may be correlated with intelligence.[1]

(d) Extension, Extrapolation: The term 'extension' as used by Burton, Campbell, Cromie, Glennon and Alderman, and in the current study referred to producing one more repetend to the immediate right of two model repetends. (Example: ABCABC___). Close and Rustigian used the same term but required the production of one more element beyond the third repetend. (Example: ABCABC____). The former task is conceptually that described by Huttenlocher (1967) and by Piaget and Inhelder (1967) as the copying of an ordered set of objects to one side of the model set[2] (Sternberg, 1973). The latter version is closer to the more general notion of extrapolation.

The term 'extrapolation' in serial pattern parlance implies the continuation to the right of a model pattern from

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1 In the case of young children, it may be a correlation with cognitive development in terms of Piagetian operations (for example, conservation, classification, seriation).

2 The difference in presentation protocols of the Burton (1973) 'extension' tasks and the copying of ordered sets would tend to yield different perceptions of the two problems.

any point in the pattern. (Example: ABCAB__).[1] This process of course demands extraction of a pattern construct[2] from within the given model.

> While a pattern can be copied in a mechanical fashion (looking at the pattern and duplicating it block by block), no such strategy will work in extending a pattern. The student must be aware of the existence of the pattern in order to extend it (McKillip, 1970, p. 17).

The extrapolation (or 'extension') process is a principal component of activities and tests involving pattern recognition. Blackall's (1975) 'continue' process, McKillip's (1970) 'extending' and Comprehensive Mathematics Inventory (Rea & Reys, 1970a) 'pattern identification', the Klahr and Wallace (1970) 'completion problems', the 'problem solving' activities of Nelson and Kirkpatrick (1975), and the Thurstone Letter Series Completion Test[3] are all examples of extrapolation. The reason for its ubiquitous nature is its

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1 This presumes the provision of sufficient elements in the model pattern to allow for rational deduction of a pattern construct.

2 "...in a series completion problem there is no absolutely right answer but children aged five to seven can solve a variety of problems of this kind producing solutions and descriptions of their solution process that agree with (a logical) strategy" (Klahr & Wallace, 1970, p. 244).

3 This is a test of adult pattern processing ability and involves relational as well as object sequences.

accurate representation of the nature of thinking (Bartlett, 1958).

> The extrapolation process is a more characteristic thinking procedure than interpolation.[1]
> ...extrapolation is more difficult than interpolation.[2]
> ...genuine thinking is always a process possessing direction (p. 33).

An extrapolation process can be intuitively transformed into an interpolation situation "by a leap to the terminal point itself and movement from this to the required additional steps" (p. 42). It is more usual to effect extrapolation through the "extraction of rules" (p. 43), "by analysis of steps of evidence and movement through additional steps to the terminal point" (p. 42).

Rustigian (1976) recorded the (problem solving) strategies of three- to five-year-olds as inferred from her observation of their overt responses to the 'extension' task. There was not sufficient evidence for systematization of the results. Blackall (1975) analyzed the errors of third grade children on her 'continue' operation applied to dual attribute (color and geometric form) serial patterns. They were adapted from the Klahr and Wallace (1970) sequences involving both color

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1 Interpolation in the traditional sense.

2 Interpolation involves a terminal point which suggests an approach but extrapolation requires the determination of the terminal point (Bartlett).

and orientation attributes. Klahr and Wallace worked with five- and six-year-olds in a year's longitudinal study of continuation tasks applied to combined color/orientation patterns. The children's strategies were analyzed and synthesized through an information processing computer program.[1]

(e) Extravariable transfer: The term 'extravariable transfer' as used by Close (1976) and in the current study, referred to the production of a model analogous[2] to a given left-right sequence[3] with respect to the pattern dimension (for example, color). As in the case of 'reproduction' this process required the copying of an ordered set of objects but with respect to only one attribute of the objects; that is, copying by analogy. (Example: red block/ blue block/red block; red house/blue house/red house). Sternberg's (1973) 'extradimensional shift' on the other hand required the identification of an analogous[4] pattern in a different dimension than the given sequence (for example: blue/red/blue/red and car/chair/ (different) car/ (different) chair). As such, it required an abstraction of the pattern

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1 Klahr and Wallace claimed that a combination of their model with that of Simon and Kotovsky (1963) seemed "to provide the best explanation of problem-solving behavior" (p. 243).

2 The objects of which the model was constructed differed in kind from those in the given sequence.

3 The physical response is precisely that of the 'reproduction' process.

4 The physical representations are analogies. The patterns which are abstracted would be (isomorphically) the same.

together with an inherent classification task (for example: specific name - peach; specific label - fruit; specific category - food).

(f) Intravariable transfer: The term 'intravariable transfer' as introduced by Campbell (1973) and used in the current study referred to the production of a model to represent the same (abstract) pattern as a given left-right sequence, in the same dimension but with different "positive and negative clues" (Sternberg, 1973, p. 6). (for example: red/blue/red/blue and green/yellow/green/yellow). Sternberg's 1973) 'intradimensional shift' had precisely the same meaning but required only an identification response. However, his patterns also involved inherent classification tasks. Success in this type of task demanded the ability to abstract the pattern from the given sequence with a minimum of perceptual support from the given sequence.

(g) Reversal: The term 'reversal' as used by Blackall (1975) referred to a two-element continuation on the left end of a given left-right sequence. Sternberg's (1973) 'reversal-shift' involved the identification of a pattern in which the "previously learned positive and negative cues of the original discrimination task are reversed" (p. 7),

(for example: ABBABB and BBABBA[1]). Both processes required symbolic (internal) representation (abstraction) of the given model and provided about the same amount of perceptual support as would occur in an intravariable transfer (intradimensional shift) task.

Hierarchies of Pattern Processes

Table 1 summarizes the hierarchies of linear pattern processes derived from recent individual research projects with young children.

According to the criteria of abstraction inherent in the concept of pattern recognition presented in this review, processes in part (a) of Table 2 require only order discrimination and one-to-one correspondence. They could be classified as 'pre-pattern' processes. Those in part (b) can be quite difficult without some ability to extract pattern. They might be classified as 'pattern-anticipatory' processes. Those in part (c) demand symbolic internalization of pattern. They can unequivocally be classified as 'pattern-recognition' processes.

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1 BBABBA is equivalent to AABAAB. Each of the patterns ABB and AAB is the reversal-shift of the other.

TABLE 1

HIERARCHIES OF LINEAR PATTERN PROCESSES

HIERARCHY[1]	FEATURES OF GIVEN SEQUENCES	SUBJECTS[2] AGE	SES	ABILITY	INVESTIGATOR
Reproduction[3]	Single attribute sequences of colored blocks, of colored square regions, and of letters X, S, O, I.	4-7	Low	Normal	Cromie (1971)
Identification		4-7	Low	Below Normal	Campbell (1973)
Extension		4-7	Low	Above, Below average	Burton (1973)
	As above plus sequence of geometrically shaped blocks and of geometric outlines	3-5	Mid to High	Average and Above	Rustigian (1976)
Extravariable transfer[4]	As for Cromie	4-7	Low	Below average Black, Hispanic, White	Close (1976)
Identification					
Interpolation					
Extension					
Original learning	Sequences required inherent classification: name, label, category (identification only)	4-7	Low	Above average, below average	Sternberg (1973)
Reverse shift					
Intra-dimensional					
Extra-dimensional shifts					
Select	Two-attribute color/shape sequences and matrices, number sequences and matrices	8-11	Mixed	Mixed	Blackall (1975)
Interpolate					
Continue					
Reverse					

TABLE 1 (Continued)

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1 Listed from easy (maximum of perceptual support) to difficult (minimum of perceptual support).

2 Perspective over the combined conclusions must take into account between-study variation in pattern tasks and in subjects.

3 The meanings of these terms for each investigator were set out in the previous section.

4 The bracket indicates no significant difference.

TABLE 2

OVERALL HIERARCHY OF PATTERN PROCESSES

	PROCESS	INVESTIGATOR[2]	DISTINGUISHING FEATURES[3]
	Order discrimination	Brain (1959)	Identification
	Order discrimination	Huttenlocker (1967)	Production
		Piaget & Inhelder (1967)	
(a)	Original learning	Sternberg (1973)	Repeated elements, ident.
	Reproduction*	Burton[4] (1973)	Production
	Copying	McKillip (1970)	Includes mixed mode
	Extravariable transfer*	Close (1976)	Analogy, different objects
	Identification*	Burton[5]	Selection to right[6]
	Reproduction	Frith (1970)	By Memory
(b)	Extension*	Burton[7]	Produce third repetend
	Extension	McKillip	Produce some repetend
	Interpolation	Close	Within third repetend
	Extension	Rustigian (1976), Close	Third repetend plus one
	Interpolation*	Blackall (1975)	Within any repetend
	Identification	Rea & Reys (1970)	Extrapolation to right
	Continuation	Blackall, Klahr & Wallace (1970)	
	Reversal	Blackall	Extrapolation to left
(c)	Reversal shift	Sternberg	Cues reversed, ident.
	Intravariable transfer*	Current study	Analogy, same variable
	Intradimensional shift	Sternberg	Analogy, different variable
	Extradimensional shift	Sternberg	

(a)[8] Processes requiring minimal logical thinking.
(b) Processes requiring some logical thinking.
(c) Processes requiring considerable logical thinking.
* Process in the current study

TABLE 2 (Continued)

1 For this listing it is assumed that all the given sequences would be of the same character; that is, the hierarchy is based strictly on the pattern processes per se.

2 The investigator is listed for purposes of detailed process specification as set out in the preceding section.

3 These features account in part for the hierarchical order relative to one or other of the adjacent processes.

4 Also Campbell (1973), Cromie (1971), and Rustigian (1976).

5 Also Campbell, Cromie, Rustigian, and Close (1976).

6 All the previous processes involve responses directly below the model.

7 Also Campbell and Cromie.

8 These groupings should be identifiable by factor analysis.

Research Hypothesis

Pattern task scores on cognitive processes will decrease as the proportion of abstract logical thinking required to perform the task increases.

Discussion: If the scores for the cognitive processes are represented as follows:

P_1 = reproduction score, P_2 = extravariable transfer score

P_3 = identification score, P_4 = extension score

P_5 = interpolation score, P_6 = intravariable transfer score

then Burton (1973), Campbell (1973), Cromie (1971), Glennon and Alderman (1970), and Rustigian (1976) have concurred that

$$P_1 > P_3 > P_4$$

Close's (1976) results could be expressed as

$$\frac{P_2 + P_3}{2} > \frac{P_4 + P_5}{2}$$

due to the process clustering he implied.

Because of the structural similarity of reproduction and extravariable transfer (visual matching of colors), it is reasonable to conjecture that P_1 and P_2 would be very close[1] but the more basic nature of the reproduction process (exact copying) would suggest that

$$P_1 > P_2$$

The conjectured proximity of P_1 and P_2 and the fact of previous separations of P_1 and P_3 suggest that

$$P_1 > P_2 > P_3$$

since Close's identification scores were not adjusted for guessing in a multiple choice situation.[2]

Because the interpolation process in the Close instrument involved insertion of elements only within the last of the three given repetends, it very closely resembled the extension process (McKillip, 1970). Hence, lack of significant difference between P_4 and P_5 was not surprising.

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1 It was such a conjecture which led Close (1976) to replace reproduction by extravariable transfer. The requirement of virtually no deductive thinking automatically placed reproduction first.

2 Also, Close tested the null hypotheses: $P_2 = P_3 = P_4 = P_5$ and was not able to derive a significant difference between P_2 and P_3 nor between P_4 and P_5. However, had the null hypotheses associated with the rank ordering $P_2 > P_3 > P_4 > P_5$ been tested it might have been rejected.

Logically, however, two adjacent complete repetends in an extension task (ABBABB---) makes for much easier identification of the nature of a repetend than does a sequence of three repetends with one element missing from one of the repetends (ABBA_BABB). Even the location of the extremities of the repetends can be a considerable challenge (especially when the subject is not told that there are always three repetends present).

Previous results and the developing argument so far have suggested the rank order: $P_1 > P_2 > P_3 > P_4 > P_5$. The additional cognitive process of intravariable transfer provides virtually no perceptual support for solution of its tasks. It requires a complete abstraction of the pattern form (Sternberg, 1973) in order to allow creation of the same pattern form but by means of another arrangement of colors. The rank order of scores hypothesized for the current study on the basis of diminishing visual support and increasing logical abstraction is: $P_1 > P_2 > P_3 > P_4 > P_5 > P_6$.

Furthermore, it would be expected that high correlations would occur among P_1, P_2, P_3 and among P_4, P_5 because of the like problem solving skills involved within each group (McKillip, 1970). Similarly, P_6 would be expected to stand on its own because of the distinctly higher level skills associated with it.

Colored one-inch wooden cubes were chosen in part because of the subjects' familiarity with them. Also activities with these blocks have an intrinsic appeal for children throughout the four-to-seven-year age range (McKillip, 1970).

Since the type of internal representation necessary for correct response to a pattern task depends on the cognitive process rather than the medium of the pattern stimulus (Newell and Simon, 1972; Bruner, 1966; Rustigian, 1976; Braine, 1959), all of the tasks were presented with the same stimulus.

The colors red, orange, yellow, green, and blue used in this study replicated those used by Burton (1973), Glennon and Alderman (1970), Campbell (1973), Cromie (1971), and Rustigian (1975). In deference to subjects with color-vision deficiencies, no combinations of red and green nor blue and green were used (Burton), (as illustrated in Table 3). (Close (1976), responded to the problem by deleting green and using only the other four colors). Care was taken to select blocks with the same finish and shading within each color and to place them with like-textured sides face up.

For each pattern task, blocks were glued to 8 inch by 14 inch sheets of heavy gauge white poster board[1] to form the appropriate repetends or partial repetends. 8 inch by 20 inch cards were used for the identification process in order to better separate the given pattern from the choice options. A twenty item reservoir consisting of four blocks in each of the five colors was provided for completion of the task in each of the reproduction, extension, interpolation, and intravariable transfer processes. A twenty item reservoir consisting of four toy plastic farm animals in each of the five colors was provided for completion of the extravariable transfer task. Not all the same color models were of the same animal.[2]

Responses were recorded on specially designed forms. (See Appendix B)

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1. See Appendix A for a sample protocol and illustrations.

2. Pilot procedures indicated that variation among animal types did not deter children in responding to the tasks.

Method:

The instrument consisted of twenty-four linear patterning tasks involving six cognitive processes (reproduction, extra-variable transfer, identification, extension, interpolation, and intravariable transfer) and four pattern forms (AB, ABB, AABB, and ABC)[1]. within each process. (See TABLE 3.)

Reproduction process: Given two repetends of a pattern, subjects will reproduce the pattern immediately below the given pattern. Example: ABBABB

Extravariable transfer process: Given two repetends of a pattern, subjects will reproduce the pattern with a different form of the same stimulus. Example: yellow block, red block, yellow block, red block as compared with yellow plastic animal, red plastic animal, yellow animal, red animal.

Identification process: Given one or two repetends of a pattern, subjects will select that pattern from among three options which include two distractors. Example: Yellow, Red, Red; choices: YYR, RRY, YRR.

Extension process: Given two repetends of a pattern, subjects will extend the pattern by one more repetend. Example: ABBABB ___.

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1. A study of pattern complexity and its effect on young children's pattern task achievement is also to be the subject of a separate paper.

Interpolation process: Given three repetends of a pattern, one of which is missing an element (in other than the first or last place of the sequence), subjects will complete the partial repetend. Example: ABBAB_ABB.

Intravariable transfer process: Given two repetends of a pattern, subjects will reproduce the pattern with other items of the same form of stimulus. Example: yellow, red, yellow, red and red, orange, red, orange.

TABLE 3

4X6 MATRIX MODEL OF THE PATTERN TASK INSTRUMENT

Cognitive Processes		Patterns and Color[1] Models			
		PT_1:AB	PT_2:ABB	PT_3:AABB	PT_4:ABC
P_1:	Reproduction	RY	YGG	LLYY	OLR
P_2:	Extravariable Transfer	RL	LYY	GGYY	YLR
P_3:	Identification	LR	ROO	GGOO	YRO
P_4:	Extension	YO	OGG	RRLL	GYO
P_5:	Interpolation	GO	LOO	OORR	RLY
P_6:	Intravariable Transfer	OR	GYY	YYLL	LOR

1 L=Blue, G=Green, O=Orange, R=Red, Y=Yellow

Each subject was tested on all four pattern tasks within each of the six cognitive processes. Because the cognitive processes were treated as repeated measures they were assigned to each subject in a random order (Kerlinger, 1973; Kirk, 1968). The four pattern tasks were also presented in a random order to each subject. A training task involving the trivial pattern (PT_0) was provided for each process in order to familiarize subjects with the procedure of the pattern tasks unique to each process. Table 4 sets out a sample order of tasks for one subject.

TABLE 4
A SAMPLE ORDER OF TASKS FOR ONE SUBJECT

Cognitive Process	Patterns (Sample random orders)				
P_1: Reproduction	PT_0	PT_2	PT_4	PT_3	PT_1
P_3: Identification	PT_0	PT_1	PT_3	PT_2	PT_4
P_4: Extension	PT_0	PT_1	PT_4	PT_2	PT_3
P_5: Interpolation	PT_0	PT_4	PT_3	PT_1	PT_2
P_6: Intravariable Transfer	PT_0	PT_3	PT_1	PT_2	PT_4
P_2: Extravariable Transfer	PT_0	PT_3	PT_2	PT_1	PT_4

Data Source

Target Population

The population investigated in this study was composed of the pupils in junior kindergarten (4400 children), senior kindergarten (5100 children), and first grade (5400 children) of a borough (population 380,000) of a Canadian metropolitan area (population 2,300,000). There were one-hundred and twenty-one elementary schools in the borough's system at the time of the investigation.

The borough is essentiallly suburban with a mix of socio-economic levels ranging from public housing developments (for low income families) through neighborhoods of modest single family housing to more fashionable townhouse and condominium developments up to executive level single family subdivisions. Latest available census figures (1971) indicated a considerable ethnic mix. While 70% of the population was from the British Isles, 90% spoke English most often in their homes. English was the language of instruction in the public school system. About 7% of the population had had some university education, 17% of the work force held professional or managerial positions, and the average total family income was $11,600.

Drawing the Sample

Two schools were selected at random from each social-class: from a low SES neighborhood, a middle SES neighborhood, and a high SES neighborhood.[1] In each school, seven subjects were chosen at random from each combination of categories: below-average ability and above-average ability students at each of the three grade levels. This selection created eighteen SES by ability by grade groups, each containing fourteen subjects. There were two-hundred and fifty-two subjects in all. Table 5 depicts the structure of the sample.

TABLE 5

STRUCTURE OF THE SAMPLE

		SOCIOECONOMIC STATUS					
		Low		Middle		High	
ABILITY LEVEL[1]		Below Avg.	Above Avg.	Below Avg.	Above Avg.	Below Avg.	Above Avg.
GRADE LEVEL	JK	S1	S4	S7	S10	S13	S16
	SK	S2	S5	S8	S11	S14	S17
	FG	S3	S6	S9	S12	S15	S18

1 Ability was not considered equivalent across the three SES levels.

1. A determination of index of neighborhood SES is in Appendix C.

To preserve subject anonymity, ranked teacher lists of potential subjects were coded with two digit numbers by the principal investigator. Subjects drawn at random were placed in the appropriate category (JA: Junior Kindergarten; Above average; JB, SA, SB, 1A, 1B) until each of the categories became filled (7 subjects). One additional subject was selected for each category to act as a spare in case of absence. To allow for these 'spares', the top quartile (above average) and bottom quartile (below average) were rounded off to the next highest number in all cases in which rounding off was necessary.

The subjects were then numerically (three digit number) coded to identify school (first digit) and grade level (e.g. JK was 01-14) without identifying teacher assessed ability level. The latter precaution was taken in order to eliminate tester bias and avoid comparison of code numbers by students with a possible parental reaction to the labelling of students. However, for purposes of analysis of data it was necessary to recode the subjects after the testing so that the middle digit indicated ability level (2,4,6 - above-average:JK,SK,FG; 1,3,5 - below-average: JK,SK,FG).

Research Design

The research design was an Ex Post Facto analog[1] of Kirk's (1968) split-plot factorial design, type SPF-pru.q with three categorical (between-block) variables (A: SES, C: ability level, and D: grade) and one within-block treatment (repeated measure) (B: linear patterning tasks). The design is specified in Table 6.

Repeated measures designs have the advantage of each subject's acting as its own control. The disadvantage lies in the cumulative effect of the repeated trials on the subjects. A subject undergoing the kth trial has been conditioned by the first (k-1) trials so that he is no longer the 'same subject'. To control for this multiple treatment effect, the treatment must be presented in a random order to each subject (Kirk, 1968; Kerlinger, 1973) (unless of course the effect of a fixed order of treatments is being studied).

The significant advantage of the repeated measures design for this study was that each subject's scores could be combined so as to be considered as a judge's ranking of the six cognitive processes (in terms of difficulty). Scores could also be combined so as to be considered as a ranking of the four pattern forms (in terms of difficulty).

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1 Kirk's (1968) split-plot designs call for experimental (manipulated) independent variables. The variables in this study (SES, grade, and ability) were not experimental.

TABLE 6

EX POST FACTO ANALOG OF A SPLIT PLOT DESIGN,
TYPE SPF-pru.q

SES LEVEL A	ABILITY LEVEL C	GRADE LEVEL D	B: REPEATED MEASURE FACTOR LINEAR PATTERNING TASK [1] b_1	b_2	b_3	. . .	b_{24}
a_1	c_1	d_1	S_1[2]	S_1	S_1	. . .	S_1
		d_2	S_2	S_2	S_2	. . .	-
		d_3	S_3	S_3	-	. . .	-
	c_2	d_1	S_4	S_4	-	. . .	-
		d_2	S_5	S_5	-	. . .	-
		d_3	S_6	-	-	. . .	-
a_2	c_1	d_1	S_7	-	-	. . .	-
		d_2	S_8	-	-	. . .	-
		d_3	S_9	-	-	. . .	-
	c_2	d_1	S_{10}	-	-	. . .	-
		d_2	S_{11}	-	-	. . .	-
		d_3	S_{12}	-	-	. . .	-
a_3	c_1	d_1	S_{13}	-	-	. . .	-
		d_2	S_{14}	-	-	. . .	-
		d_3	S_{15}	-	-	. . .	-
	c_2	d_1	S_{16}	-	-	. . .	-
		d_2	S_{17}	-	-	. . .	-
		d_3	S_{18}	-	-	. . .	-

TABLE 6 (Continued)

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1 The pattern task scores can be grouped and summed as

$$\sum_{k=1}^{4} b_k, \quad \sum_{k=5}^{8} b_k, \quad \sum_{k=9}^{12} b_k, \quad \sum_{k=13}^{16} b_k, \quad \sum_{k=17}^{20} b_k, \quad \sum_{k=21}^{24} b_k,$$

in order to yield respectively the six cognitive process scores: P_1, P_2, P_3, P_4, P_5, P_6 (as defined in TABLE 3 as repeated measures).

2 See Table 5 for the specific coding of the subject groups S1, ..., S18.

Analysis of Data

A ranking-by-judge interpretation of the repeated measures design allowed testing of the hypothesized rank order of the cognitive process (with respect to degree of abstraction) by means of the L-test (Page, 1963). For each subject group (and combinations of groups up to the full sample) the rankings of subjects were interpreted as a random sample of all possible rankings by the target 'population'. Hence results of the rejection of appropriate null hypotheses were generalizable to the respective target sub-populations.

Hierarchy of Pattern Processes

Each subject's process scores were ranked as though he were a judge of pattern-process hierarchy. The ranks for each process were then summed over all subjects. Table 7 indicates the results of the calculations for the L-test of the hypothesized rank order of process scores (Page, 1963). (H_1: $P_1=P_2=P_3=P_4=P_5=P_6$ against the ordered alternative H_1: $P_1>P_2>P_3>P_4>P_5>P_6$). Hence, H_1 is rejected for the whole sample in favour of the hypothesized hierarchy of pattern processes.

Table 8 summarizes the L-test results for the secondary null hypotheses relating to the hierarchy of pattern processes. Since the L-statistic is significant at the $p < .001$ level for each subject group S1 through S18 (which are pairwise disjoint), L is also significant at the $p < .001$ level for all combinations (union) of two or more of groups S1 through S18 (Ridge, 1976).

TABLE 7

L-TEST: HIERARCHY OF PATTERN PROCESSES (ALL SUBJECTS)

NUMBER OF RANKINGS	REPRODUCTION	EXTRA-VARIABLE TRANSFER	IDENTIFICATION	EXTENSION	INTERPOLATION	INTRA-VARIABLE TRANSFER			
	1	2	3	4	5	6	L	r	X^2_L
222	529.5[1]	553.5	582	836	1036.5	1118.5			
	529.5	1107[2]	1746	3344	5182.5	6711	18620[3]	.593[4]	390.1***[5]

*** $p < .001$

1 Sum of judges' ranks.

2 Hypothesized hierarchy ranking x sum of judges' ranks (2 x 553.5 = 1107).

3 Sum of the above products

4 P. 35 Footnote 2 - first sentence only.

5 The table of "Critical Values of L" (Page, 1963, pp. 220-223) does not include values of L for more than fifty judges. In such cases, the statistics X^2_L is used "which is distributed approximately as chi-square with 1 degree of freedom" (ibid., p. 224) where the probability in the chi-square table should be halved for the usual one-sided test.

$$X^2_L = [12L - 3mn\ (n + 1)^2]^{\ 2}/mn^2\ (n^2 - 1)\ (n + 1)$$

In this case m = 222 (the number of rankings) and n = 6 (the number of pattern processes ranked).

TABLE

L-TEST: PATTERN PROCESS HIERARCHY BGROUPS)

GROUP CODE	SAMPLE GROUP SES	ABILITY	GRADE	m[1]	L	r	R/F[3]
S1	Low	Bel.Av.[4]	JK[5]	13	1038***	.363	R
S2			SK	14	1115***	.351	R
S3			FG	13	1128***	.758	R
S4		Ab. Av.	JK	11	945***	.709	R
S5			SK	13	1142***	.820	R
S6			FG	14	1127***	.400	R
S7	Middle	Bel.Av.	JK	13	1067***	.490	R
S8			SK	10	801***	.377	R
S9			FG	12	1024***	.676	R
S10		Ab. Av.	JK	13	1122***	.731	R
S11			SK	14	1163***	.547	R
S12			FG	14	1145***	.473	R
S13	High	Bel.Av.	JK	14	1163***	.547	R
S14			SK	8	702***	.814	R
S15			FG	11	939***	.678	R
S16		Ab. Av.	JK	13	1130***	.767	R
S17			SK	11	937***	.668	R
S18			FG	12	968***	.410	R

1 Number of rankings (judges).

2 The average correlation of individual rankings with the a prior (Hypothesized) ranking. $r = 12^{L}/m \ (n^3 - n) - 3(n + 1)/(n - 1)$ (Page, 1963, p. 226). In this case n = 6 (the number of pattern process rankings) and m (the number of rankings) varies among the groups. (r values permit ready comparison of the ordering relationship among the groups).

3 Reject null hypothesis or fail to reject null hypothesis.

4 Bel.Av. = Below Average, Ab.Av. = Above Average.

5 JK = Junior Kindergarten, SK = Senior Kindergarten, FG = First Grade.

Factor analysis of process scores

The six cognitive process scores (P_1, P_2, P_3, P_4, P_5, P_6) were subjected to a factor analysis[2] for the purpose of construct validation of the instrument. The results are shown in Table 9 .

TABLE 9

FACTOR ANALYSIS OF PROCESS SCORES

PATTERN PROCESS	ROTATED FACTOR LOADINGS[1] 1	2	3	COMMUNALITY
P_1 (Reproduction)	0.870	0.110	-0.015	0.770
P_2 (Extravariable Transfer)	0.761	0.132	-0.088	0.604
P_3 (Identification)	0.733	0.274	-0.147	0.633
P_4 (Extension)	0.402	0.672	-0.185	0.647
P_5 (Interpolation)	0.089	0.913	-0.096	0.851
P_6 (Intravariable Transfer)	0.113	0.172	-0.976	0.994
SUM OF SQUARES	2.055	1.419	1.025	4.499

1 Because more than one factor was being sought, a rotation of the principal component factors was necessary, (Armor & Couch, p. 83).

2. Data - Text (Armor & Couch, 1972) factor analysis consists of the method of "principal components (Kerlinger, 1973, p. 667)" and the "Varimax (Armor & Couch, p. 86)" method of orthogonal rotations.

The negative loadings of Factor 3 mean only "that low scores on (those) variables are correlated with the factor (Armor & Couch, 1972, p. 86)". If process scores are truncated* so that all scores less than 3 are set equal to zero (thereby removing all variance in the lower scores) the factor analysis on process scores yields Table 10. Factor 3 loadings are approximately equivalent to those in Table 9 but being positive, now correlate with the higher process scores since.

TABLE 10

FACTOR ANALYSIS OF TRUNCATED* PROCESS SCORES

PROCESS* SCORES	ROTATED FACTOR LOADINGS 1	2	3	COMMUNALITY
P^*_1	0.861	0.115	0.011	0.755
P^*_2	0.740	0.119	0.079	0.568
P^*_3	0.719	0.210	0.135	0.579
P^*_4	0.414	0.679	0.131	0.650
P^*_5	0.062	0.911	0.092	0.843
P^*_6	0.101	0.144	0.982	0.994
SUM OF SQUARES	1.992	1.384	1.014	4.389

Factor 1 loads most heavily on P_1 (reproduction) hich was nonsidered theoretically to depart little from order discrimination. It also loads heavily but slightly less on P_2 (extravariable transfer) and on P_3 (identification) both of which were conjectured to depart from order discrimination only fractionally more than P_1.

As well, this factor loads significantly[1] on P_4 (extension). P_4 tasks required extension of the given sequence of two repetends by precisely one repetend. It was argued theoretically that such tasks could be considered as order discrimination to the right of the model (Piaget & Inhelder, 1967). The possible requirement of a preliminary strategy for P_4 (as reflected by the loading of Factor 2 on this process) would tend to reduce the weighing of Factor 1 which is clearly a "pre-pattern" factor. The term "pre-pattern" was introduced and discussed in conjunction with Table 2. Factor 1 could eqully well be labelled "one-to-one correspondence", "order dsicrimination", or "order strategy".

Factor 2 loads most heavily on P_5 (interpolation). P_5 tasks required the filling of a single element gap at a random location within a given sequence of three

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1 Kerlinger (1973) suggested a loading of at least .35 as "substantial" (p. 680) while Armor & Couch (1972) used the same term for loadings "over .4 or .5" (P. 86).

repetends. It was argued theoretically that such tasks could be accomplished by a "fairness" strategy or that they might even involve a "reversal" strategy if the gap were close to the left end of the sequence. Because single attribute patterns lend themselves to completion by a fairness strategy especially when the given sequences involve only full repetends,[1] it would seem to be 'solution-by-fairness-strategy' which is being expressed by Factor 2. The lesser but highly significant loading of Factor 2 on P_4 (extension) corroborates this conclusion. If extension were not accomplished by a one-to-one strategy or an order discrimination strategy then a preliminary fairness strategy would at least indicate which elements to use, if not their proper order. Order discrimination could then be employed to suggest which element must be next to the given terminal element[2]. Hence Factor 2 could be labelled "fairness strategy".[3]

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1 Balancing the numbers of elements by attribute value (not by pattern) would readily yield an acceptable solution to AB_ABCABC. Such would not be the case for AB_ABCABCA since there are 4 A's, 3 B's, and 2 C's. Should the A's and B's have equal representation or the B's and C's?

2 A fiarness strategy applied to ABCABC_____ at least indicates that one each of A, B, and C should be used. "A next to C" or "C followed by A" from the given sequence would suggest the position for A etc.

3 Although extension tasks (and extrapolation generally) lend themselves to solution by a "template strategy" (Klahr & Wallace, 1970, p. 245), such a strategy would tend to be less useful for interpolation involving a randomly located single space gap. Because of the relative weightings of Factor 2 on P_3 and P_4 it would be quite inappropriate to label this factor "template strategy".

The term "pattern anticipatory" introduced in conjunction with Table 2, is only somewhat appropriate for Factor 2. The fairness strategy is based on a pattern of numbers of elements with discriminable attribute values such as different colors but not on the positional recurrence of the stimuli. Hence, the strategy makes use of the fairness strategy is a necessary step in the development of the recognition of linear patterns or whether it serves as a distractor[1] in this development requires investigation.

Factor 3 loads significantly only on P_6 (intravariable transfer) and hence could be labelled "pattern-recognition" or "symbolic representation" or "logical thinking" in light of the earlier theoretical discussion.

Table 9 involves two other points of interest regarding Factor 3 and process P_6. One is that the sum of squares for Factor 3 is very close to 1.0 (1.025). Armor and Couch (1972) have suggested that such factors should not be interpreted since they account for no more variance than a single variable (p. 84). However, the theoretical analysis of processes (Table 2) indicated that only one of the six processes would belong in such a category. Expectation for the variance (sum of squares)

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1 Success with the fairness strategy in many early patterning problems may deter children from developing a spatial patterning strategy.

due to such a factor to be any greater would be unreasonable.

The other point lies in the 'communality' column. Each entry is the sum of the squares of the loadings of the three factors on each variable. Hence, the communality of a variable is the variance which that variable has in common with the other five variables.[1] Process P_6 (intravariable transfer) shares the greatest variance with the other processes. This might be interpreted to imply that the (logical) thinking required to succeed in P_6 tasks could be applied to all the other processes (which of course, is the case).

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1. Communality is also called "common factor variance (Kerlinger, 1973, p. 471)".

Validity

Further consideration of Table 9 of factor loadings yields a measure of validity[1] of the six process instrument. The total communality (sum of the variable communalities or sum of the factor sums of squares) is the common factor variance of the instrument or its validity component (Kerlinger, 1973). "Since the total variance of a set of n variables in a correlation matrix is always n" (Armor & Couch, 1972, p. 84), the total variance in this case is 6. Hence the validity of the six process instrument is 4.4991/6 or .75.

The factor analysis of the truncated scores (Table 10) yields an instrument communality of 4.389 and a subsequent validity of 4.389/6 or .73. Thus, setting all scores below three equal to zero reduced the validity of the instrument only by 2 percentage points. The implication is that scores of 1 or 2 out of a possible process score of 4 were not indicative of 'knowledge' of the process.

Construct Validation

Previous analysis of theory and of the research literature led to the conjecture that three constructs

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1 Kerlinger (1973) has discussed a variance definition of test validity as "...the proportion of the total variance of a measure that is common factor variance" (p. 469).

underlay the six process instrument. The occurrence of the three factors in the current factor analysis does not really constitute a factor validation of the instrument for two reasons. The first is that the conjectured Factor 1 was not expected to load significantly on process P_4 (extension). The fact that both of the "discovered" (Kerlinger, 1973, p. 687) Factor 1 and Factor 2 did load on P_4 somewhat changed the anticipated complexion of Factor 2 from "pattern-anticipatory" to "fairness strategy". Discovered Factor 3 did concur with its conjectured counterpart.

Secondly, to validate the instrument now means to validate the discovered factors. This procedure calls for (a) the formation of a hypothesis relating to the currently inferred nature of these factors, (b) the construction of new tests and their administration to new samples of subjects, and (c) factor analysis of the subsequent scores.

> This will certainly not end the matter. One will have to test, among other things, the factors' relation to other factors. One will have to place the factors, as constructs, in a nomological network of constructs[1] (Kerlinger, 1973, p. 687).

Item Validity

"To the extent that an item measures the same thing as the total score does, to that extent the item is valid" (Kerlinger, 1973, p. 468). The significant ($p < .001$)

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1 That is, the network of constructs will have to satisfy some accepted physical laws or rules of reasoning.

correlation of items (a) with their respective process scores and (b) with the total score (Table 11) indicate satisfactory item validity within each subtest (process) and for the instrument overall. Table 11 also lists an index of difficulty and an index of discriminability for each item.

TABLE 11

INSTRUMENT ITEM ANALYSIS

PATTERN PROCESS	TEST ITEM	CORRELATION WITH PROCESS SCORE	CORRELATION WITH TOTAL SCORE	INDEX OF DIFFICULTY (MEAN ITEM SCORE)[1]	INDEX OF DISCRIMINATION $(p_u - p_l)^2$
P_1 Reproduction	1	0.879***	0.591***	0.841	0.305
	2	0.910***	0.623***	0.846	0.270
	3	0.910***	0.636***	0.846	0.287
	4	0.932***	0.680***	0.837	0.322
P_2 Extravariable Transfer	5	0.899***	0.629***	0.841	0.322
	6	0.941***	0.654***	0.833	0.3[illegible]
	7	0.921***	0.592***	0.824	0.29[illegible]
	8	0.909***	0.621***	0.824	0.297
P_3 Identification[3]	9	0.733***	0.465***	0.833	0.240
	10	0.726***	0.585***	0.850	0.291
	11	0.600***	0.419***	0.833	0.249
	12	0.733***	0.516***	0.833	0.266
P_4 Extension[4]	13	0.874***	0.679***	0.533	0.701
	14	0.894***	0.695***	0.480	0.720
	15	0.897***	0.692***	0.480	0.720
	16	0.844***	0.644***	0.463	0.644
P_5 Interpolation	17	0.657***	0.399***	0.449	0.349
	18	0.712***	0.489***	0.586	0.397
	19	0.707***	0.467***	0.335	0.384
	20	0.634***	0.382***	0.427	0.323

TABLE 11 (Continued)

PATTERN PROCESS	TEST ITEM	CORRELATION WITH PROCESS SCORE	CORRELATION WITH TOTAL SCORE	INDEX OF DIFFICULTY (MEAN ITEM SCORE)[1]	INDEX OF DISCRIMINATION $(p_u - p_l)$[2]
P_6 Intravariable Transfer	21	0.855***	0.480***	0.207	0.201
	22	0.878***	0.472***	0.132	0.237
	23	0.756***	0.364***	0.084	0.144
	24	0.905***	0.461***	0.123	0.220

*** $p < .001$

1 If an item is scored 0 or 1, the simplest index of difficulty is the mean item score. "This score also serves as an average index of difficulty for individuals" (Guilford, 1954, p. 418).

2 "The simplest index....is the difference $p_u - p_l$ where P_u and P_l are proportions of [successful subjects] in upper and lower groups [50%], respectively" (Guilford, 1954, p. 425).

3 The lower item validity within P_3 and P_5 with respect to their respective process scores again reflects the multiple choice response problem dealt with under RELIABILITY. Average item difficulty based on (raw) item scores shows 'identification' (P_3) to be easier than extravariable transfer (P_2). Such an inference would be unfounded.

4 Extension tasks had the greatest item validity with respect to the total score criterion.

Instrument Reliability

Hoyt's test of reliability (Guilford, 1954) by means of repeated measures analysis of variance was performed on the individual task sxores (a) for the whole instrument and (b) for each cognitive process in order to determine (a) the reliability of the instrument and (b) the reliability of each of its process subtests (Kerlinger, 1973). The reliability (r_{tt}) so determined is equivalent to the Kuder-Richardson formula 20 method and to Cronbach's α applied to individual test items (Guilford).

Table 12sets out the reliability (a) of the instrument as a twenty-four item test, (b) of each of the processes as a four item subtest, and (c) each of the patterns as a six item subtest.

TABLE 12

INSTRUMENT RELIABILITY

Part of Instrument	Reliability Coefficient (r_{tt})[1]
Whole Test	0.90
Process Subtests	
Reproduction	0.93
Extravariable Transfer	0.94
Identification	0.65
Extension	0.91
Interpolation	0.61
Intravariable Transfer	0.86

1 $r_{tt} = \frac{V_e}{V_{ind}}$ where V_e is the residual mean square and V_{ind} the individual (unit) mean square in a repated measures analysis of variance on item individual pattern scores (Kerlinger, 1973, p. 448). The anova results are in TABLE 13.

TABLE 13

ANALYSIS OF VARIANCE TABLES FOR TESTS OF RELIABILITY

PART OF INSTRUMENT	SOURCE	SUM OF SQUARES	DF	MEAN SQUARE	RELIABILITY COEFFICIENT (r_{tt})
Whole	Tasks	389.257	23	16.924	$1 - \frac{0.123}{1.249}$
	Residual	640.371	5198	0.123	
	Individuals	282.269	226	1.249	$= 0.90$
Reproduction	Tasks	0.017	3	0.006	$1 - \frac{0.031}{0.429}$
	Residual	21.483	702	0.031	
	Individuals	100.441	234	0.429	$= 0.93$
Extra-Variable Transfer	Tasks	0.026	3	0.009	$1 - \frac{0.030}{0.478}$
	Residual	20.974	702	0.030	
	Individuals	111.767	234	0.478	$= 0.94$
Identification	Tasks	0.051	3	0.017	$1 - \frac{0.092}{0.263}$
	Residual	65.449	708	0.092	
	Individuals	62.129	236	0.263	$= 0.65$
Extension	Tasks	0.740	3	0.247	$1 - \frac{0.072}{0.794}$
	Residual	50.510	705	0.072	
	Individuals	186.664	235	0.794	$= 0.91$
Interpolation	Tasks	7.826	3	2.609	$1 - \frac{0.173}{0.439}$
	Residual	122.424	708	0.173	
	Individuals	103.560	236	0.439	$= 0.61$
Intra-Variable Transfer	Tasks	1.767	3	0.589	$1 - \frac{0.044}{0.327}$
	Residual	30.233	693	0.044	
	Individuals	75.431	231	0.327	$= 0.86$

The reliability coefficient for IDENTIFICATION (0.65) is relatively low due to the multiple choice nature of its responses. Each subject selected a pattern from among three possibilities (the correct response and two distractors). There was a 33 1/3% chance of a correct answer on the basis of a random selection. For purposes of further analysis of scores, IDENTIFICATION (total) scores (P_3) have been corrected as follows:

$$P_3' = P_3 - 1/3\ (4 - P_3), \quad P_3 > 0$$

$$P_3' = P_3, \quad P_3 = 0$$

which is the standard correction procedure for guessing with respect to multiple choice testing items.

The reliability coefficient for INTERPOLATION (0.61) is relatively low due to the multiple choice nature of its responses. Each subject selected the color of the single filler block from among five possible colors. On the basis of a similar argument, for purposes of further analysis of scores, INTERPOLATION (total) scores (P_5) have been corrected as follows:

$$P_5' = P_5 - 1/5\ (4 - P_5), \quad P_5 > 0$$

Figure 1 illustrates the better spread among the process scores with the adjustments to P_3 and P_5 scores effected.

The overall test reliability of 0.90 can be interpreted

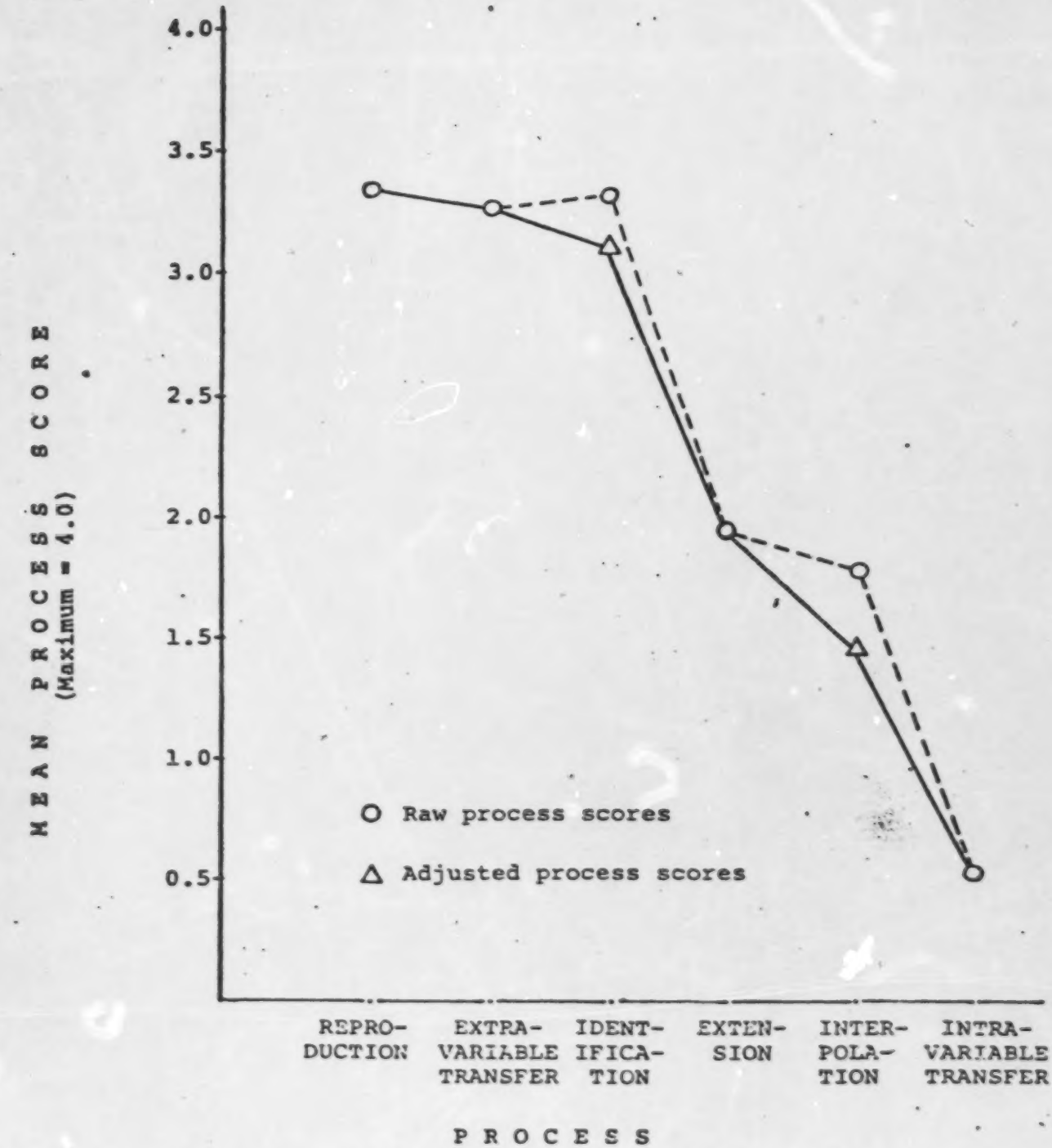

FIGURE 1. RAW AND ADJUSTED MEAN PROCESS SCORES

1 Adjustments to P_3 and P_5 raw scores were described under Reliability.

as an indicator of high stability[1] in the instrument (Kerlinger, 1973). That is, if the test were administered to the same subjects more than once, the rank order of the individuals would remain the same in 90% of the cases.

The reason that the process subtest reliabilities are substantial (less the cited exceptions) is the high intercorrelation among the patterns. Each subtest contains only four items but the items are very close to being equivalent.

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1 An alternative interpretation is "internal consistency" (Kerlinger, 1973, p. 451). That is, any random subtest of the instrument would yield the same rank order of individuals 90% of the time.

TABLE 14

SUMMARY OF DATA ANALYSIS

PROCEDURE	CONCEPT ANALYZED	FINDINGS
1. L-test of process score ranks	Hierarchy of pattern processes	P_1, P_2, P_3, P_4, P_5, P_6
2. Factor Analysis of Process scores	Constructs underlying Pattern Processes	Three factors: 'order strategy', 'fairness strategy', 'logical thinking'.
3. Repeated measures ANOVA on individual task scores	Reliability of Instrument and Subtests	Instrument reliability: 0.90 Process score adjustment (P_3, P_5) based on subtest reliabilities.
4. Instrument item Analysis	Index of difficulty, index of discrimination	EXTENSION (P_4) tasks are by far the best discriminators.

Conclusions and Discussion

This study has considered the meaning of the term 'pattern recognition' as it applies to young children four through seven years of age. The responses of the children to linear pattern tasks have served as a means of investigation of the sequence of cognitive steps in the natural development of logical thinking. The instrument has provided an arena for the exercising of both visual and symbolic thinking as manifested in the strategies brought to the patterning problems.

Pattern Process Hierarchy

Analysis of the ranks of the process scores has unequivocally established the hierarchy of pattern processes to be REPRODUCTION (P_1), EXTRAVARIABLE TRANSFER (P_2), IDENTIFICATION (P_3); EXTENSION (P_4), INTERPOLATION (P_5), and INTRAVARIABLE TRANSFER (P_6). This sequence was hypothesized on the basis of a combination of empirical evidence and theoretical consideration of the degree of pattern abstraction required for the various tasks. Meaningful testing of the hypotheses relating to degree of abstraction by means of subject scores depended on the assumption that the difficulty of these processes (as reflected by the process scores) for young children (either at the intuitive stage of the preoperational level or in transition to the concrete operational stage)

lay in direct proportion to the amount of logical thinking necessary to accomplish the tasks (all other aspects being equal). The assumption was justified empirically by factor analysis of the process scores.[1] The factors were identified in terms of types of reasoning which require successively more symbolic thinking. (Factor 1: order discrimination; Factor 2: fairness strategy; Factor 3: (abstract) pattern recognition). Factor 1 loaded (substantially) on P_1, P_2, P_3, and P_4 (in that order), Factor 2 loaded on P_4 and P_5 (in that order), while Factor 3 loaded only on P_6.

The hierarchy just established is instrument specific in the following sense. Reference should be made to Table 2 and to the related discussion immediately preceding it so that the current results may not be mistaken for corroboration or refutation of other findings in the field in which similar terminology appears.

The real significance of such strong hierarchical evidence lies not so much in the order of the processes themselves but in the implied sequential development of the thought processes necessary to cope with particular cognitive tasks.

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1 It should be noted that the results of a factor analysis do not depend on process scores (i.e., order of difficulty). Factor analysis depends on the correlations among scores in order to identify underlying common constructs.

Significance of the Study

The inclusion of linear pattern tasks in early childhood curricula and diagnostic procedures is a well established fact (as evidenced by earlier numerous citations). That the great preponderance of these tasks involve extrapolation (especially) or interpolation processes is to the credit of their designers. It is extrapolation in particular which encompasses the essence of the thinking process as discussed in conjunction with Bartlett's (1958) work. Also, the best discriminators in the current instrument were the extension tasks (Table 11) followed by the interpolation tasks.

The educational assets of linear pattern tasks for young children and their teachers are many-fold. Of initial importance is the play-orientation of the materials which house an intrinsic appeal for youngsters (McKillip, 1970). Rustigian (1976) found that young children really do enjoy participating in patterning activities.

Secondly, patterning activities can be organized to run the gamut from true play discovery-type situations through structured fixed-response items to open-ended pattern design tasks in one or more dimensions.

Early childhood teachers can derive considerable insight into young children's thought processes through astute observation of youngsters at play with materials which lend themselves to patterning activities. Some youngsters may of their own accord enter into pattern design. Such manifestation would certainly indicate youngster readiness to be confronted with more formal pattern tasks. Highly structured fixed-response activities (as exemplified in the current instrument) permit teacher insight into the problem solving strategies available to young children. The important implication of such activities lies not so much in a child's accomplishing certain tasks but in what thought processes he may have brought to bear on the problem.

The teacher's routine does not permit extended interviewing of individual youngsters and computer matching of responses (as in the Klahr and Wallace (1970) study) in order to assess the nature of each child's thinking. A minimum of such knowledge however is necessary and of concern for effective teachers so that they might better respond to individual needs through what has come to be called 'cognitive fit' (Kagan, 1970; Skemp, 1971; Wapner, 1969). However, monitoring of children's non-verbal responses within a hierarchy of pattern tasks can provide evidence of the level of internalization. Suydam and Weaver (1971) through their review of literature have stressed that children of all ability levels who receive learning tasks appropriately graded to their levels of achievement retain and transfer equally well to new situations of appropriate difficulty.

A third asset of linear patterning activities is their role as pre-number experiences (McKillip, 1970) and as early language arts experiences (Sternberg, 1974).[1] The current study does not deal with these aspects of patterning except to emphasize the topological order concept (Piaget & Inhelder, 1967) inherent in patterning activities associated with 'Factor 1'. Patterning activities restricted to this

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1 Sternberg (1975) has stated that success with 'pattern reversals' involves the same high level of thinking required to deal with number and number operations. Blackall (1975) found successful pattern processing at the third grade level was related to high mathematics achievement.

domain could hardly be considered to have a number orientation. They may serve however in a 'pre-geometry' context.

The involvement of number concepts could arise through implementation of the fairness strategy to solve linear pattern tasks,[1] but such a strategy should be considered at best as an interim stage in pattern recognition. It makes no use of the spatial orientation of the pattern stimuli, but involves consideration only of the numbers of like attribute values and the balancing of same. This is a situation in which acuity for or predisposition with number could be detrimental to the development of more mature spatial patterning processes. It is such a situation against which teachers should guard. Youngsters who are consistently successful with Factor 2 types of patterning processes (such as extrapolation and interpolation) should soon be introduced to Factor 3 processes (such as intra-variable transfer) which require symbolic internalization. It may be necessary virtually to wean some youngsters from a penchant for the 'fairness strategy'. Encouragement of flexibility in approaches to problem solving is highly advised (Blackall, 1975).

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1 McKillip (1970) introduced number into pattern extension problems through the use of relational patterns (for example:)

The extent to which children should be 'taught' patterning processes is a question which parallels the on-going debate among devotees of Piaget as to whether 'conservation' for instance should, or indeed can, be taught to non-conservers. The implication of the current study is that it would be futile to try to instruct youngsters in solving patterning problems which demand a level of internalization which has yet to be developed. It would seem that children's ability or lack of same with regard to particular categories of tasks would serve as a signal of their levels of internalization. The teacher's role would involve an ongoing provision of a hierarchy of tasks, success in which would signal developing levels of thinking.

Peel's (1971) admonition that over emphasis on mental ability could lead to relative neglect of the changes possible from good teaching indicates a rather ostrich-like stance. Surely teaching cannot help but improve when teachers are able through routine methodology to assess more accurately the strengths of their charges, especially in the matter of thinking processes. Also the recent findings of Martin (1975) regarding the genetic implications of subject specific abilities lay all the more emphasis on the importance of mental abilities.

Recommendations for future research:

Now that this descriptive research has established the feasibility of the existence of constructs underlying various cognitive pattern processes, some experimental research in the area might be appropriate. A very straight-forward experiment but one which would yield immediate classroom ramifications could be based on treatements which involve various stimulus/response combinations on the same set of pattern tasks. Possible combinations are set out in Table 15.

TABLE 15

PATTERN TASK STIMULUS/RESPONSE COMBINATIONS

Mode of Presentation of Pattern Stimulus	Mode of Subject Response
1. Administrator builds model sequence in front of subject.	Subject produces response.
2. Administrator builds model sequence in front of subject.	Subject identifies response.
3. Administrator presents subject with complete model of the sequence.	Subject produces response.
4. Administrator presents subject with complete model of the sequence.	Subject identifies response.

A more involved experiment could center on the notion of teachability of patterning ability.[1] The treatment would consist of games and activities based on linear patterning and designed in deference to the hierarchy of cognitive processes established in the current study. It would also include opportunity for and encouragement of subject creation of pattern. The treatment would be carried out by a principal investigator in a club format of school activity so that subjects for both control and treatment groups could be chosen at random within participating schools. The experiment could be carried out according to the Solomon Four Group design (Campbell & Stanley, 1963) in which there are two treatment groups and two control groups, one of each of which would be assigned randomly to a pre-test and all of which would be post-tested. None of the testing would be done by the principal investigator.

There has been considerable discussion throughout about the 'fairness strategy' (Klahr & Wallace, 1970). Is it a necessary step in the ontogeny of pattern recognition? Is it a distractor from that development? Does it even involve a construct distinct from spatial pattern processing

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1 Because this concept was theoretically disclaimed earlier in the chapter does not preclude its serving as the subject of experimental investigation.

but parallel to it and related to number patterning? The fairness strategy appears to work so well in so many linear patterning situations that it may deter development of more general, spatially oriented processing strategies. Some insight into this dilemma might be provided by the following type of experiment. The treatment group would be given pattern tasks in order pairs such as the filling of the gaps in

1. AB_ABCABC

and 2. AB_ABCABCA

where the fairness-strategy assists in solving number 1, but would tend to confuse in number 2. The control group would be assigned only the type of task exemplified by number 2. Results should indicate the extent to which tasks suitable to fairness-strategy solution assist with or detract from the solution of linear patterning problems which do not lend themselves to fairness-strategy solution.

Summary:

The key conclusions of the study are:

1. Three strategy-oriented constructs underlie linear pattern task performance by children ages four through seven:[1] 'order strategy', 'fairness strategy', and 'pattern recognition'. Success in pattern tasks related to particular strategies serves as a guide to knowledge of the levels of thinking available to these children.
2. REPRODUCTION, EXTRAVARIABLE TRANSFER, IDENTIFICATION, EXTENSION, INTERPOLATION, AND INTRAVARIABLE TRANSFER is the hierarchy of cognitive pattern processes of the current instrument for children ages four through seven.

The main educational implications arising from the study involve:

(a) the extensive scope of problem solving abilities for which patterning tasks can be designed,

(b) the insight into levels of thinking available to youngsters which can be derived from their responses to a hierarchy of pattern tasks (in deference to the 'cognitive fit' position),

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1 It is to be understood that the children referred to are those of the target population in the community studied and in similar communities.

(c) a caution regarding over-use of the fairness strategy, and

(d) the possible futility of trying to teach successively higher level patterning skills.

The recommendations for future research encompass experiments dealing with

(a) various combinations of stimulus/response styles,

(b) the teachability of patterning strategies, and

(c) the role of the fairness strategy in the development of patterning ability.

APPENDIX A

LINEAR PATTERN TASKS
PROTOCOL

Process P1: Reproduction
Training Task

TASK: Given a model pattern and a reservoir of blocks, the tester correctly selects and places blocks from the reservoir to form a pattern that matches the model. The subject is then required to produce the same model.

DIRECTIONS:

1. Tester says LOOK AT THIS (point).
   I AM MAKING THIS (point) HERE (point).
2. Tester correctly places block in left-most unfilled space while saying THIS GOES HERE (NEXT).
3. Continue 2. until pattern is complete.
4. Tester points to the model then to the construction while saying ARE THEY THE SAME?
5. Tester nods to indicate "yes" and says YES, THEY ARE THE SAME.
6. Tester then replaces all the blocks in the reservoir.
7. Tester says LOOK AT THIS (point).
   MAKE THIS (point) HERE (point).
8. If the subject does not respond, repeat 7. once.
9. If the subject correctly selects and places a block and all spaces are not filled, then tester (if subject does not continue) points to the unfilled space(s) while saying WHAT GOES NEXT?

10. If the subject does not respond, stops, or incorrectly selects or places a block, then the tester repeats the entire procedure only once. Go to 1.

11. If the subject successfully matches the model with a construction, go to the next task.

LINEAR PATTERN TASKS
PROTOCOL

Process P1: Reproduction
Patterns: 1-4

TASK: Given a model pattern of blocks, the subject selects and places blocks from the reservoir to form a pattern that matches the model.

DIRECTIONS:

1. Tester says LOOK AT THIS (point).

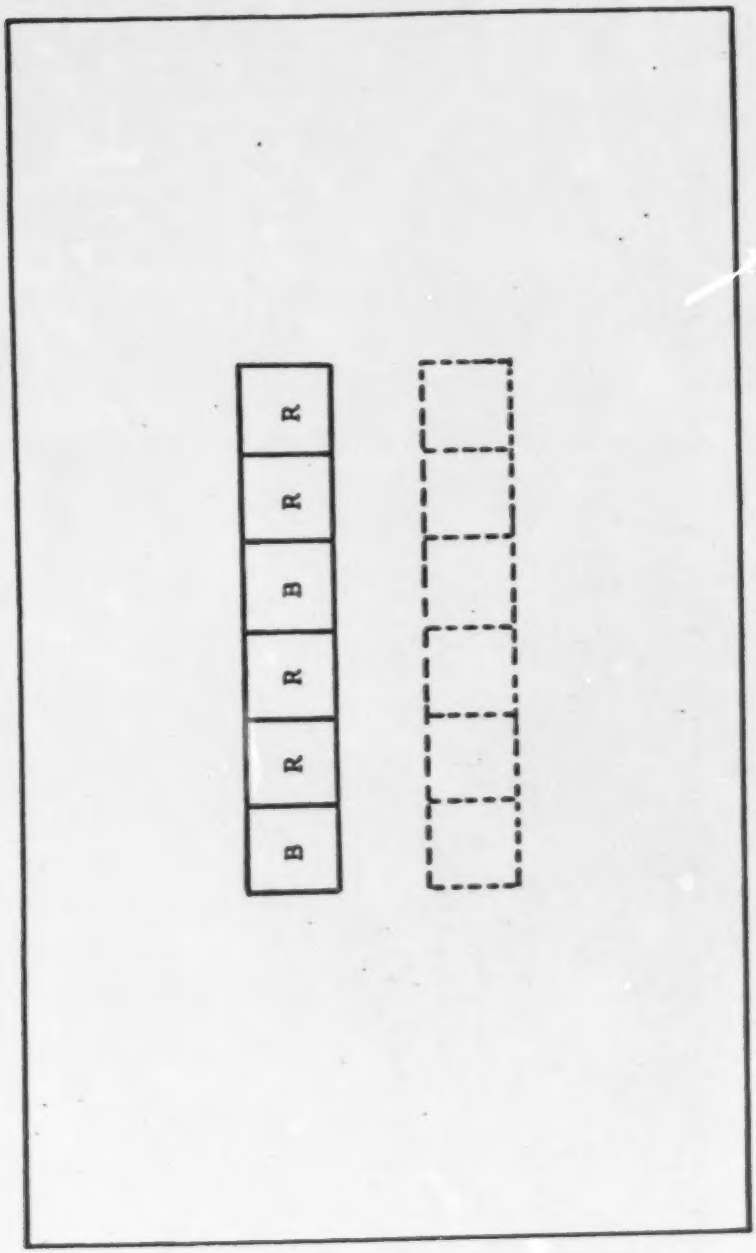
   MAKE THIS (point) HERE (point).

2. If the subject does not respond, repeat 1. once.

3. Allow the subject a reasonable amount of time to finish. Tester marks the recording sheet for Process P1.

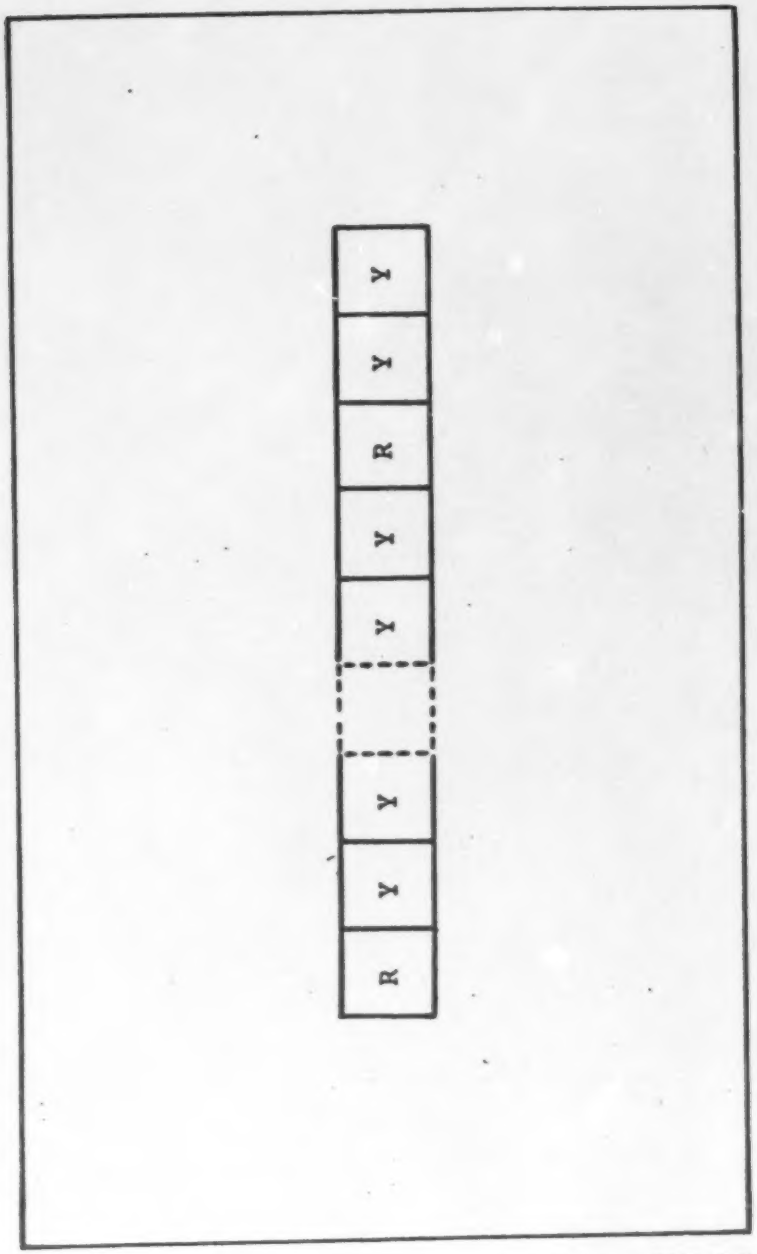
   Go to the next task.

PATTERN TASK MODEL
PROCESS: REPRODUCTION
EXTRAVARIABLE TRANSFER
INTRAVARIABLE TRANSFER



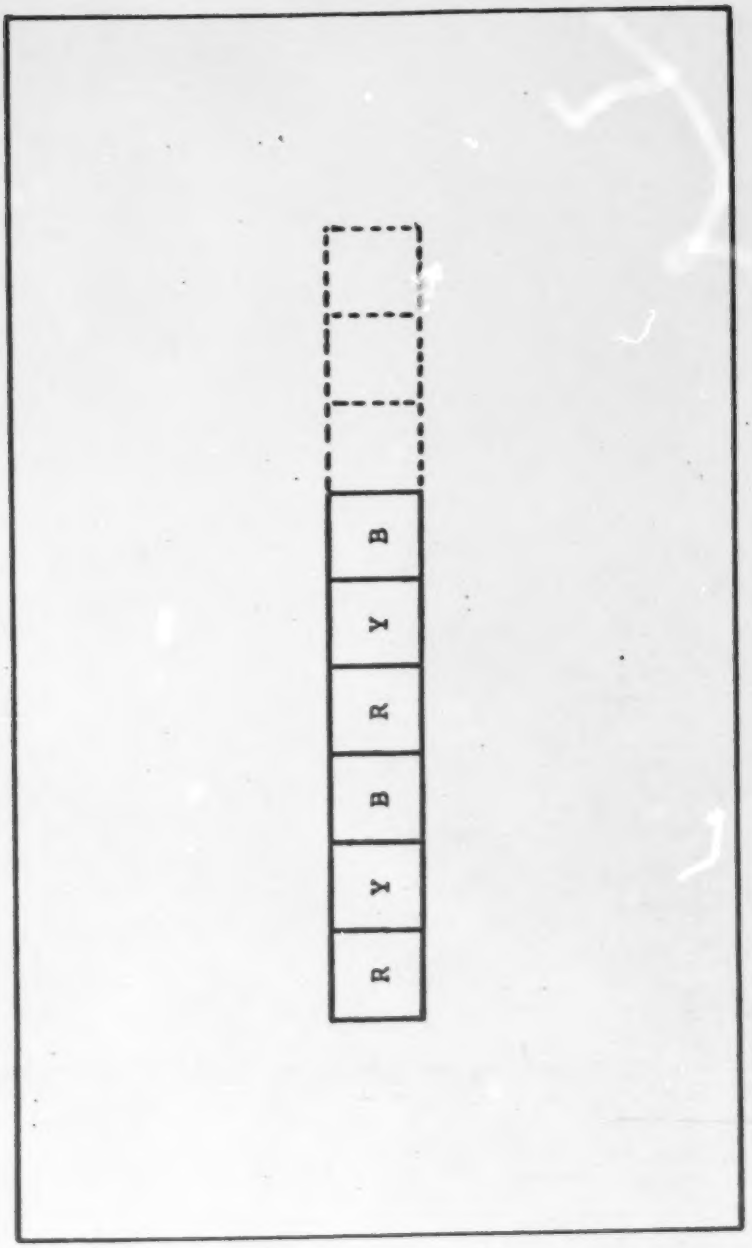
Scale 1/2":1"

PATTERN TASK MODEL
PROCESS: INTERPOLATION



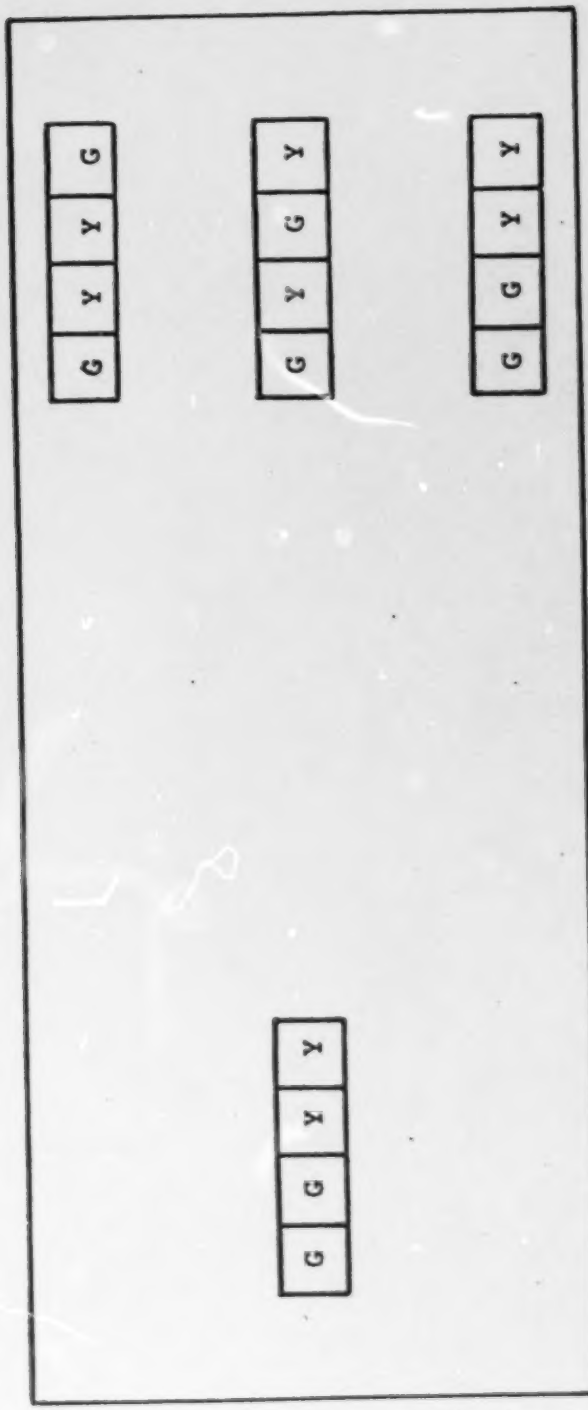
Scale 1/2":1"

PATTERN TASK MODEL
PROCESS: EXTENSION



Scale 1/2":1"

PATTERN TASK MODEL
PROCESS: IDENTIFICATION



Scale 1 cm:1"

APPENDIX B

PATTERN INVESTIGATION SCORE SHEET

Tester: Set up Program of Processes and Patterns Before Meeting Subjects

SUBJ. NUMBER	SCHOOL	GRADE	ABILITY	TESTER	TOTAL SCORE	ANALYSIS NUMBER

1st NO. PROCESS ____ _____

PATTERN (P_1,P_2,P_3,P_4)	RESPONSE	SCORE (1/0)
	TOTAL	

2nd NO. PROCESS ____ _____

PATTERN (P_1,P_2,P_3,P_4)	RESPONSE	SCORE (1/0)
	TOTAL	

3rd NO. PROCESS ____ _____

PATTERN (P_1,P_2,P_3,P_4)	RESPONSE	SCORE (1/0)
	TOTAL	

4th NO. PROCESS ____ _____

PATTERN (P_1,P_2,P_3,P_4)	RESPONSE	SCORE (1/0)
	TOTAL	

5th NO. PROCESS ____ _____

PATTERN (P_1,P_2,P_3,P_4)	RESPONSE	SCORE (1/0)
	TOTAL	

6th NO. PROCESS ____ _____

PATTERN (P_1,P_2,P_3,P_4)	RESPONSE	SCORE (1/0)
	TOTAL	

APPENDIX C

DETERMINATION OF INDEX OF NEIGHBORHOOD SES

The Warner Scale (Warner, Meeker & Eels, 1965) is based on four criteria: occupational group (Table K1), source of income (Table K2), house (apartment) type (Table K3), nature of dwelling area (Table K4). The scale is designed for application to individuals. In this study census data and direct observation have been used to derive an index for each neighborhood based on neighborhood means.

TABLE K1

RELATION OF CENSUS AND WARNER
OCCUPATIONAL GROUPS

Census[1] Group	Description	Approximate Warner Rating (1-7)
11	Managerial, Administrative and Related Occupations	2
27	Teaching and Related Occupations	2.5
31	Occupations in Medicine and Health	2
21	Occupations in Natural Sciences, Engineering and Mathematics	2
23	Occupations in Social Sciences and Related Fields	2.5
25	Occupations in Religion	2.5
33	Artistic, Literary, Recreational and Related Occupations	3
41	Clerical and Related Occupations	3.5
51	Sales Occupations	4
61	Service Occupations	4.5
71	Farming, Horticultural and Animal Husbandry Occupations	5
73	Fishing, Hunting, Trapping and Related Occupations	5
75	Forestry and Logging Occupations	5
77	Mining and Quarrying including Oil and Gas Field Occupations	5
81/82	Processing Occupations	6
83	Machining and Related Occupations	6
85	Product Fabricating, Assembling and Repairing Occupations	6
87	Construction Trades Occupations	6
91	Transport Equipment Operating Occupations	6

1 Canadian Census, 1971.

TABLE K2

SOURCE OF INCOME RATING

Rating	Source of Income
1	Inherited 'wealth'
2	Earned 'wealth'
3	Profits and fees (professionals)
4	Salary (and commission)
5	Wages
6	Private relief
7	Public relief

TABLE K3

HOUSE (APARTMENT[1]) - TYPE RATING

Rating	House - Type
1	Excellent - very large single family home in good repair - element of ostentation - landscaped grounds
2	Very good - not quite the quality of (1) but larger than utility demands
3	Good - slightly larger than utility demands - less ostentatious - more conventional
4	Average - 1 1/2 and 2 storey wood, frame, and brick - conventional- good lawns - no landscaping
5	Fair - not in as good condition as 'average'
6	Poor - rundown
7	Very poor - almost uninhabitable

1 Apartments are judged by exterior condition and size of the living units.

TABLE K4

DWELLING AREA[1] RATING

Rating	Nature of Dwelling Area
1	Very high - where the social elite live
2	High (fewer mansions and pretentious homes than in (1) but its rating is largely a matter of reputation
3	Above average - nice but not pretentious ('Society' doesn't live there)
4	Average - workingmen's homes (unpretentious but neat)
5	Below average - close to factories, business section or railway
6	Low - rundown and semi-slums
7	Slums

1 "Members of the same [social] class tend to live in the same part of town" (Warner, Meeker & Eels, 1965, p. 152).

TABLE K5

SES INDEX CALCULATION

	Weighting	Neighborhoods 1	2	3
Occupation Group	4	4.25	3.35	3.47
Source of Income	3	4.04	3.84	3.85
House-Type Rating	3	4.68	4.30	2.80
Dwelling Area Rating	2	4.84	3.70	3.80
SES Index (Weighted Sum)		52.84	45.22	39.43

For this study, neighborhood (1) was called 'low SES', neighborhood (2) was called 'middle SES', and neighborhood (3) was called 'high SES'. (According to the Warner Scale, these neighborhoods range from 'upper-lower' class to 'lower-middle' class).

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